

AP Calculus AB Stuff You Must Know

Algebra Stuff	Trig Stuff Identities																																				
<p>Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$</p> <p>Point-slope form: $y - y_0 = m(x - x_0)$</p> <p>Standard Form: $Ax + By = C$</p> <p>Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p> <p>Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p> <p>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p> <p>$\ln(e) = 1$ $\ln 1 = 0$ $\ln(ab) = \ln(a) + \ln(b)$ $\ln(a/b) = \ln(a) - \ln(b)$</p> <p>$e^x > 0$ $e^{2+x} = e^2 e^x$ $e^{-x} = \frac{1}{e^x}$</p>	<p>$\sin(2x) = 2 \sin x \cos x$</p> <p>$\cos(2x) = \cos^2 x - \sin^2 x$</p> <p>$\cos(2x) = 2 \cos^2 x - 1$</p> <p>$\cos(2x) = 1 - 2 \sin^2 x$</p> <p>$\sin^2 x = \frac{1 - \cos 2x}{2}$</p> <p>$\cos^2 x = \frac{1 + \cos 2x}{2}$</p> <p>$\sin^2 x + \cos^2 x = 1$</p> <p>$1 + \tan^2 x = \sec^2 x$</p> <p>$1 + \cot^2 x = \csc^2 x$</p> <p>$\sec x = \frac{1}{\cos x}$</p> <p>$\csc x = \frac{1}{\sin x}$</p> <p>$\sin(-x) = -\sin(x)$ <i>Odd</i></p> <p>$\cos(-x) = \cos(x)$ <i>Even</i></p> <p>$\tan(-x) = -\tan(x)$ <i>Odd</i></p> <p>$\cot(-x) = -\cot(x)$ <i>Odd</i></p> <p>$\sec(-x) = \sec(x)$ <i>Even</i></p> <p>$\csc(-x) = -\csc(x)$ <i>Odd</i></p>																																				
<table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th colspan="4">Trig Values</th> </tr> <tr> <th>θ</th> <th>$\sin \theta$</th> <th>$\cos \theta$</th> <th>$\tan \theta$</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>$\pi/6$</td> <td>$1/2$</td> <td>$\sqrt{3}/2$</td> <td>$\frac{\sqrt{3}}{3}$</td> </tr> <tr> <td>$\pi/4$</td> <td>$\sqrt{2}/2$</td> <td>$\sqrt{2}/2$</td> <td>1</td> </tr> <tr> <td>$\pi/3$</td> <td>$\sqrt{3}/2$</td> <td>$1/2$</td> <td>$\sqrt{3}$</td> </tr> <tr> <td>$\pi/2$</td> <td>1</td> <td>0</td> <td>DNE</td> </tr> <tr> <td>π</td> <td>0</td> <td>-1</td> <td>0</td> </tr> <tr> <td>$3\pi/2$</td> <td>-1</td> <td>0</td> <td>DNE</td> </tr> </tbody> </table>	Trig Values				θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	0	0	1	0	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\frac{\sqrt{3}}{3}$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\pi/2$	1	0	DNE	π	0	-1	0	$3\pi/2$	-1	0	DNE	
Trig Values																																					
θ	$\sin \theta$	$\cos \theta$	$\tan \theta$																																		
0	0	1	0																																		
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\frac{\sqrt{3}}{3}$																																		
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1																																		
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$																																		
$\pi/2$	1	0	DNE																																		
π	0	-1	0																																		
$3\pi/2$	-1	0	DNE																																		
ASTC to assign +/-																																					

Differential Calculus Formulas and Rules

$$\frac{d}{dx}(x)^n = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arc cot} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arc sec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{arccsc} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Applications of the first and second derivative

Curve Sketching

- To find a critical value, set $f'(x) = 0$ or undefined
- Use a sign chart to determine if the function has a relative extrema. Make sure you write sentences summarizing the results.
- Use can also use the Second Derivative Test to verify extrema. Suppose that x_0 is a critical value. If $f''(x_0) < 0$, then x_0 is the x-coordinate of the relative maximum. If $f''(x_0) > 0$, then x_0 is the x-coordinate of the relative minimum.
- To find points of inflection, set $f''(x) = 0$ or undefined. Then, show that the sign of $f''(x)$ changes as x passes through that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval $[a, b]$ and y is some value between $f(a)$ and $f(b)$, then there exists at least one value $x = c$ in the open interval (a, b) where $f(c) = y$.

In other words, a continuous function must pass through every y -value between $f(a)$ and $f(b)$.

Mean Value Theorem

If a function, $f(x)$ is continuous on a closed interval $[a, b]$ AND is differentiable on an open interval (a, b) , then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = \frac{f(b) - f(a)}{b - a}$.

In other words, there is at least one point on a smooth curve where the tangent line can be drawn so that it is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If a function, $f(x)$ is continuous on a closed interval $[a, b]$ AND is differentiable on an open interval (a, b) AND $f(a) = f(b)$, then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = 0$.

In other words, if the endpoints of a differentiable function have the same y -coordinates, there is at least one point inside the interval where the slope of the tangent line is equal to zero. This is a special case of the

Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln |\cos x| + c$$

$$\int \cot x dx = \ln |\sin x| + c$$

$$\int \sec x dx = \ln |\sec x + \tan x| + c$$

$$\int \csc x dx = -\ln |\csc x + \cot x| + c$$

$$\int \sec^2 x dx = \tan x + c$$

$$\int \csc^2 x dx = -\cot x + c$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\int \csc x \cot x dx = -\csc x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int \frac{1}{1+x^2} dx = \arctan x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arcsec} x + c$$

(Integration by parts)

Fundamental Theorem of Calculus – Part 1	Fundamental Theorem of Calculus – Part 2	Average Value Theorem
$\int_a^b f'(x) dx = f(b) - f(a)$	$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$	If a function $f(x)$ is continuous on the closed interval $[a, b]$, then there exists some number $x_0 = c$ such that $f(c) = \frac{1}{b-a} \int_a^b f(x) dx$
Volume of a Solid of Revolution (disk method) $V = \pi \int_a^b \left((OR)^2 - (IR)^2 \right) dx \text{ or } dy$	Volume of a Solid with a Known Cross-Section $V = \int_a^b \text{Area}(x) dx$	
Particle Motion Formulas		
$\text{velocity} = \frac{d}{dt} (\text{position})$	$\text{acceleration} = \frac{d}{dt} (\text{velocity})$	$\text{displacement} = \int_{t_1}^{t_2} v(t) dt$
$\text{total distance} = \int_{t_1}^{t_2} v(t) dt$	$\text{Avg velocity} = \frac{\text{position}_2 - \text{position}_1}{\text{time}_2 - \text{time}_1}$	Final Position = $x(t_1) + \int_{t_1}^{t_2} v(t) dt$